

- **6. Total differential form**

$$f(x) = x^2 \rightarrow df = f'(x) dx \rightarrow df = 2x dx$$

- Let f be a function with two variables x and y , differentiable on its domain of definition (its partial derivatives exist).
- Total differential of the function f at the point (x_0, y_0) :
- $df(x_0, y_0) = f'_x(x_0, y_0)dx + f'_y(x_0, y_0)dy$.
- Then, the total differential of the function f is given by:
- $df = f'_x dx + f'_y dy$.

- **Example.** Determine the total differential form of $f(x, y) = x^2 + y$

we have $df = f'_x dx + f'_y dy = 2x dx + dy$ at the pt $(1, 2)$

$$df|_{(1, 2)} = 2 \times 1 dx + dy = 2 dx + dy$$

$$\left\{ \begin{array}{l} f'_x(x, y) = 2x + 0 \\ \quad \quad \quad = 2x \\ f'_y(x, y) = 0 + 1 \\ \quad \quad \quad = 1 \end{array} \right.$$

- **7. Extrema Without Constraints.**

$$f'(x) = 0$$

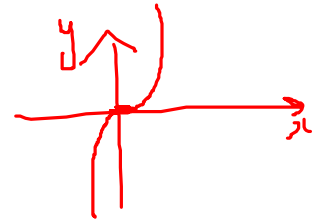
- Let $f(x, y)$ a two-variable function. Our goal is to look for the point (x_0, y_0) Where $f(x_0, y_0)$ the maximum or minimum value that f can reach. It should be noted that extrema (minimum or maximum) may not exist.
- Step 1: Let's look for the critical points, in other words the points that are the candidates to be extrema.
- Definition a point $A(x_0, y_0)$ is said to be a critical point if these coordinates verify the following system:
 - $$\begin{cases} f'_x(x_0, y_0) = 0 \\ f'_y(x_0, y_0) = 0 \end{cases}$$

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- **7. Extrema Without Constraints.**

- Step 2: Let's calculate $\Delta(x_0, y_0)$ Where $\Delta(x, y)$ is given by :

- $\Delta(x, y) = \begin{vmatrix} f''_{xx} & f''_{yx} \\ f''_{xy} & f''_{yy} \end{vmatrix} = f''_{xx} \times f''_{yy} - f''_{xy} \times f''_{yx}$
Hessian



- Step 3: Interpretation :

- If $\Delta(x_0, y_0) = 0$ then we can't conclude anything about the point $A(x_0, y_0)$.
- If $\Delta(x_0, y_0) < 0$ then point $A(x_0, y_0)$ is a saddle point. (Not an extremum)
- If $\Delta(x_0, y_0) > 0$ then we have to check the sign of $f''_{xx}(x_0, y_0)$ ($f''_{yy}(x_0, y_0)$)
 - If $f''_{xx}(x_0, y_0) > 0$ Then we have a minimum relative to f and this minimum is $f(x_0, y_0)$
 - Si $f''_{xx}(x_0, y_0) < 0$ Then we have a maximum relative to f and that maximum est $f(x_0, y_0)$

- **Example.** Find critical points and extremas of the following function when they exist:

- $f(x, y) = x^2 + y^2$

1st step:
 $f'_x = 0 \Rightarrow 2x + 0 = 0 \Rightarrow 2x = 0 \Rightarrow x = 0 \rightarrow$ the critical pt is $O(0,0)$

- **Solution.**

- Step 1: Search for critical points.

- Let's solve the following system:

- $$\begin{cases} f'_x(x, y) = 0 \\ f'_y(x, y) = 0 \end{cases} \Rightarrow \begin{cases} 2x = 0 \\ 2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

- Then, the only critical point is $O(0,0)$.

- Step 2:

- $\Delta(x, y) = \begin{vmatrix} f''_{xx} & f''_{yx} \\ f''_{xy} & f''_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$

$$\begin{cases} f'_x = 2x \\ f'_y = 2y \end{cases}$$

- Thus $\Delta(0,0) = 4 > 0$. In this case it is necessary to check $f''_{xx}(0,0)$.

- $f''_{xx}(0,0) = 2 > 0$

- As a result, we have a minimum relative to f and that minimum is $f(0,0) = 0^2 + 0^2 = 0$